J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

FYJC FINAL EXAM - 02

DURATION - 3 HR MARKS - 80

SOLUTION SET

SECTION - I

Q1. (A) Attempt ANY **SIX OF** THE FOLLOWING (12)

Q1

01. Differentiate the following function with respect to x

$$9x^2 - 2\sqrt{x} + 4\log x - 7^x + 25$$

SOLUTION :

$$y = 9x^2 - 2\sqrt{x} + 4\log x - 7^x + 25$$

$$\frac{dy}{dx} = 18x - 2\frac{1}{2\sqrt{x}} + \frac{4}{x} - 7^x \log 7$$

$$= 18x - \frac{1}{\sqrt{x}} + \frac{4}{x} - 7^x \log 7$$

02. Find k if the area of the triangle whose vertices are A (4 , k) ; B (–5 , –7); C(–4 , 1) is 38 sq. units

SOLUTION :

$$A\ (\Delta ABC) = 38$$

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 38$$

$$\begin{vmatrix} 4 & k & 1 \\ -5 & -7 & 1 \\ -4 & 1 & 1 \end{vmatrix} = \pm 76$$

$$4(-7-1) - k(-5+4) + 1(-5-28) = \pm 76$$

$$4(-8) - k(-1) + 1(-33) = \pm 76$$

$$-32 + k - 33 = \pm 76$$

$$-65 + k = \pm 76$$

$$-65 + k = 76 \quad \text{OR} \quad -65 + k = -76$$

$$k = 141 \qquad\qquad k = -11$$

03. find equation of circle with radius 5 and concentric with circle $x^2 + y^2 + 4x - 6y = 0$

SOLUTION

STEP 1 : $x^2 + y^2 + 4x - 6y = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = 4 \;;\; 2f = -6$

$g = 2 \;;\; f = -3 \;;\; c = 0$

$C \equiv (-g, -f)$

$\equiv (-2, 3)$

STEP 2 : $C(-2,3)$, $r = 5$

$(x - h)^2 + (y - k)^2 = r^2$

$(x + 2)^2 + (y - 3)^2 = (5)^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 25$

$x^2 + y^2 + 4x - 6y + 13 - 25 = 0$

$x^2 + y^2 + 4x - 6y - 12 = 0$ .... equation of the circle

04. **Evaluate** $\lim\limits_{x \to -4} \dfrac{x^2 + 3x - 4}{x^2 + 9x + 20}$

SOLUTION

$$\lim_{x \to -4} \frac{x^2 + 3x - 4}{x^2 + 9x + 20}$$

$$= \lim_{x \to -4} \frac{(x + 4)(x - 1)}{(x + 4)(x + 5)} \qquad x \to -4 \;;\; x \neq -4 \quad \therefore x + 4 \neq 0$$

$$= \lim_{x \to -4} \frac{x - 1}{x + 5}$$

$$= \frac{-4 - 1}{-4 + 5}$$

$$= -5$$

05. $\tan^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{1}{4}\right) = \frac{\pi}{4}$

SOLUTION

$$= \tan^{-1}\left(\frac{\frac{3}{5} + \frac{1}{4}}{1 - \frac{3}{5} \cdot \frac{1}{4}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{12 + 5}{20}}{\frac{20 - 3}{20}}\right)$$

$$= \tan^{-1}\left(\frac{17}{17}\right)$$

$$= \tan^{-1}(1)$$

$$= \pi/4$$

06. Find equation of parabola having focus at (3,0) and directrix $x + 3 = 0$

SOLUTION :

Let the equation of the parabola : $y^2 = 4ax$

Focus : $S \equiv (a, 0) \equiv (3,0)$ .... given $\therefore a = 3$

Directrix : $x + a = 0$

$x + 3 = 0$ .... Given $\therefore a = 3$

Hence equation of the parabola : $y^2 = 4ax$

$y^2 = 4(3)x$

$y^2 = 12x$

07. Find $\frac{dy}{dx}$ if $y = (4x^2 - 7x + 5) . \sec x$

SOLUTION :

$$\frac{dy}{dx} = (4x^2 - 7x + 5) \frac{d}{dx}\sec x + \sec x \frac{d}{dx}(4x^2 - 7x + 5)$$

$$= (4x^2 - 7x + 5) \sec x.\tan x + \sec x \ (8x - 7)$$

$$= \sec x \left[(4x^2 - 7x + 5) .\tan x + 8x - 7\right]$$

08. Evaluate : $\tan(-495°)$

SOLUTION :

$\tan(-495°)$

$= -\tan(495°)$

$= -\tan(540° - 45°)$

$= -\tan(180° \times 3 - 45°)$

$= - \ (-\tan 45)$

$= 1$

Q2. (A) Attempt ANY **TWO OF** THE FOLLOWING (06)

Q2A

01. Prove $\sqrt{\dfrac{1 + \sin\theta}{1 - \sin\theta}} = \tan\left(\dfrac{\pi}{4} + \dfrac{\theta}{2}\right)$

SOLUTION :

$$\text{LHS} = \sqrt{\frac{\cos^2 \theta/2 + \sin^2 \theta/2 + 2\sin \theta/2 . \cos \theta/2}{\cos^2 \theta/2 + \sin^2 \theta/2 - 2\sin \theta/2 . \cos \theta/2}}$$

$$= \sqrt{\frac{(\cos \theta/2 + \sin \theta/2)^2}{(\cos \theta/2 - \sin \theta/2)^2}}$$

$$= \frac{\cos \theta/2 + \sin \theta/2}{\cos \theta/2 - \sin \theta/2}$$

Dividing Numerator & Denominator by $\cos \theta/2$

$$= \frac{\dfrac{\cos \theta/2 + \sin \theta/2}{\cos \theta/2}}{\dfrac{\cos \theta/2 - \sin \theta/2}{\cos \theta/2}}$$

$$= \frac{1 + \tan \theta/2}{1 - \tan \theta/2} = \text{RHS}$$

$$\text{RHS} = \tan(\pi/4 + \theta/2)$$

$$= \frac{\tan \pi/4 + \tan \theta/2}{1 - \tan \pi/4 \tan \theta/2}$$

$$= \frac{1 + \tan \theta/2}{1 - \tan \theta/2}$$

LHS = RHS

02. Prove : $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$

SOLUTION :

$$= \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{5}}{1 - \frac{1}{2} \cdot \frac{1}{5}}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left(\frac{\frac{5+2}{10}}{\frac{10-1}{10}}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left(\frac{7}{9}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left(\frac{\frac{7}{9} + \frac{1}{8}}{1 - \frac{7}{9} \cdot \frac{1}{8}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{56+9}{72}}{\frac{72-7}{72}}\right)$$

$$= \tan^{-1}\left(\frac{65}{65}\right)$$

$$= \tan^{-1}(1) = \pi/4 = \text{RHS}$$

03. Prove : $\sin(\theta - \pi/6) + \cos(\theta - \pi/3) = \sqrt{3} \cdot \sin\theta$

SOLUTION :

$$(\sin\theta\cos\pi/6 - \cos\theta.\sin\pi/6) + (\cos\theta\cos\pi/3 + \sin\theta.\sin\pi/3)$$

$$= (\sqrt{3}/2 .\sin\theta - 1/2\cos\theta) + (1/2\cos\theta + \sqrt{3}/2 .\sin\theta)$$

$$= \sqrt{3}/2 .\sin\theta + \sqrt{3}/2 .\sin\theta$$

$$= \sqrt{3} \cdot \sin\theta$$

Q2. (B) Attempt ANY **TWO OF** THE FOLLOWING (08)

01. Find equation of hyperbola in the standard form whose eccentricity = √2 & distance between foci = 8√2

Q2B

SOLUTION :

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

distance between foci $= 8\sqrt{2}$

$$2ae = 8\sqrt{2}$$

$$ae = 4\sqrt{2}$$

subs $e = \sqrt{2}$ $\quad a\sqrt{2} = 4\sqrt{2}$

$$a = 4$$

Now ; $b^2 = a^2(e^2 - 1)$

$b^2 = 16\ (2 - 1)$

$b^2 = 16$

$\therefore$ the equation of the hyperbola is

$$\frac{x^2}{16} - \frac{y^2}{16} = 1$$

02. Find eccentricity , coordinates of foci , equation of directrices , length of major and minor axes and length of latus rectum for **$3x^2 + 4y^2 = 1$**

SOLUTION :

$$\frac{x^2}{1/3} + \frac{y^2}{1/4} = 1$$

$$a^2 = 1/3 \quad \therefore \; a = 1/\sqrt{3}$$

$$b^2 = 1/4 \quad \therefore \; b = 1/2 \qquad a > b$$

Eccentricity

$$b^2 = a^2(1 - e^2)$$

$$\frac{1}{4} = \frac{1}{3}(1 - e^2)$$

$$\frac{3}{4} = 1 - e^2$$

$$e^2 = 1 - \frac{3}{4}$$

$$e^2 = \frac{1}{4}$$

✓ $e = 1/2$

$$ae = 1/2 \times 1/\sqrt{3} = 1/2\sqrt{3}$$

$$\frac{a}{e} = \frac{1/\sqrt{3}}{1/2} = 2/\sqrt{3}$$

✓ foci $\equiv (\pm ae \; , 0) \equiv (\pm 1/2\sqrt{3} \; , 0)$

✓ eq. of directrices : $x = \pm\, a/e$

$x = \pm\, 2/\sqrt{3}$

✓ length of major axis $= 2a = 2/\sqrt{3}$

✓ length of minor axis $= 2b = 1$

✓ length of latus rectum $= \dfrac{2b^2}{a} = \dfrac{2(1/4)}{1/\sqrt{3}}$

$$= \frac{1/2}{1/\sqrt{3}}$$

$$= \sqrt{3}/2$$

03. find equ. of circle concentric with $x^2 + y^2 - 2x - 6y - 7 = 0$ and area 616 sq. units

SOLUTION

STEP 1 :

$x^2 + y^2 - 2x - 6y - 7 = 0$

On comparing with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -2$; $2f = -6$

$g = -1$; $f = -3$; $c = 0$

$C \equiv (-g, -f) \equiv (1,3)$

STEP 2 :

area = 616

$\pi r^2 = 616$

$r^2 = \frac{616}{\pi}$

$r^2 = \frac{616 \times 7}{22} = 196$

$r = 14$

STEP 3 :

$C(1,3)$, $r = 14$

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 1)^2 + (y - 3)^2 = (14)^2$

$x^2 - 2x + 1 + y^2 - 6y + 9 = 196$

$x^2 + y^2 - 2x - 6y + 10 - 196 = 0$

$x^2 + y^2 - 2x - 6y - 186 = 0$ …….. equation of the circle

Q3. (A) Attempt ANY **TWO OF** THE FOLLOWING (06)

Q3A

01. if $f(x) = \dfrac{x+1}{x-1}$ and $g(x) = \dfrac{2x+3}{3x-2}$, find fog

SOLUTION : $fog = f(g(x))$

$$= \frac{g(x)+1}{g(x)-1}$$

$$= \frac{\left(\dfrac{2x+3}{3x-2}\right)+1}{\left(\dfrac{2x+3}{3x-2}\right)-1}$$

$$= \frac{\dfrac{2x+3+3x-2}{3x-2}}{\dfrac{2x+3-3x+2}{3x-2}}$$

$$= \frac{5x+1}{5-x}$$

02. Solve using Cramer's Rule : $x - y + z = 4$, $2x + y - 3z = 0$, $x + y + z = 2$

SOLUTION :

$$D = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} \end{array} \begin{aligned} &= 1(1+3)+1(2+3)+1(2-1) \\ &= 1(4) + 1(5) + 1(1) \\ &= 4 + 5 + 1 \quad = 10 \end{aligned}$$

$$Dx = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 4 & -1 & 1 \\ 0 & 1 & -3 \\ 2 & 1 & 1 \end{vmatrix} \end{array} \begin{aligned} &= 4(1+3)+1(0+6)+1(0-2) \\ &= 4(4) + 1(6) + 1(-2) \\ &= 16 + 6 - 2 \quad = 20 \end{aligned}$$

$$Dy = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 1 & 4 & 1 \\ 2 & 0 & -3 \\ 1 & 2 & 1 \end{vmatrix} \end{array} \begin{aligned} &= 1(0+6)-4(2+3)+1(4-0) \\ &= 1(6) - 4(5) + 1(4) \\ &= 6 - 20 + 4 \quad = -10 \end{aligned}$$

$$Dz = \begin{vmatrix} 1 & -1 & 4 \\ 2 & 1 & 0 \\ 1 & 1 & 2 \end{vmatrix} \begin{aligned} &= 1(2-0)+1(4-0)+4(2-1) \\ &= 1(2) + 1(4) + 4(1) \\ &= 2 + 4 + 4 \\ &= 10 \end{aligned}$$

$$x = \frac{Dx}{D} \; ; \; y = \frac{Dy}{D} \; ; \; z = \frac{Dz}{D}$$

$$= \frac{20}{10} \qquad = \frac{-10}{10} \qquad = \frac{10}{10}$$

$$= 2 \qquad = -1 \qquad = 1$$

SS : $\{2,-1,1\}$

03. $y = \dfrac{2 + 3.\cos x}{3 + 2.\cos x}$. Find dy/dx

SOLUTION :

$$\frac{dy}{dx} = \frac{(3 + 2.\cos x)\frac{d}{dx}(2 + 3.\cos x) - (2 + 3.\cos x)\frac{d}{dx}(3 + 2.\cos x)}{(3 + 2.\cos x)^2}$$

$$= \frac{(3 + 2.\cos x).\left[0 + 3(-\sin x)\right] - (2 + 3.\cos x)\left[0 + 2(-\sin x)\right]}{(3 + 2.\cos x)^2}$$

$$= \frac{(3 + 2.\cos x)(-3.\sin x) - (2 + 3.\cos x).(-2.\sin x)}{(3 + 2.\cos x)^2}$$

$$= \frac{-3.\sin x\,(3 + 2.\cos x) + 2.\sin x\,(2 + 3.\cos x)}{(3 + 2.\cos x)^2}$$

$$= \frac{-9.\sin x - 6.\sin x.\cos x + 4.\sin x + 6.\sin x.\cos x}{(3 + 2.\cos x)^2}$$

$$= \frac{-5.\sin x}{(3 + 2.\cos x)^2}$$

Q3. (B) Attempt ANY **TWO OF** THE FOLLOWING (08)

Q3B

01. Evaluate : $\lim_{x \to 0} \dfrac{e^x + e^{-x} - 2}{\cos 3x - \cos 5x}$

SOLUTION :

$$= \lim_{x \to 0} \frac{e^x + \frac{1}{e^x} - 2}{-2 \sin\left(\frac{3x+5x}{2}\right) . \sin\left(\frac{3x-5x}{2}\right)}$$

$$= \lim_{x \to 0} \frac{\frac{(e^x)^2 + 1 - 2.e^x}{e^x}}{-\ 2 \sin 4x \ .\ \sin(-x)}$$

$$= \lim_{x \to 0} \frac{\frac{(e^x - 1)^2}{e^x}}{2 \sin 4x \ .\ \sin x}$$

Dividing Numerator & Denominator by x^2 , $x \to 0$, $x \neq 0$

$$= \lim_{x \to 0} \frac{\frac{(e^x - 1)^2}{e^x . x^2}}{\frac{2 \sin 4x \ .\ \sin x}{x^2}}$$

$$= \lim_{x \to 0} \frac{\frac{(e^x - 1)^2}{x^2} \ \frac{1}{e^x}}{2 \frac{\sin 4x}{x} . \frac{\sin x}{x}}$$

$$= \lim_{x \to 0} \frac{\left(\frac{e^x - 1}{x}\right)^2 \ \frac{1}{e^x}}{2.4 \frac{\sin 4x}{4x} . \frac{\sin x}{x}}$$

$$= \frac{(\log e)^2 . \frac{1}{e^0}}{2.4.(1).(1)}$$

$$= \frac{1}{8}$$

02. the demand function is given as $P = 175 + 9D + 25D^2$

Find the total revenue and marginal revenue when demand is 10

SOLUTION :

Total Revenue

$R = pD$

$= (175 + 9D + 25D^2).D$

$= 175D + 9D^2 + 25D^3$

Put $D = 10$

$= 1750 + 900 + 25000$

$= 27650$

Marginal Revenue when D = 10

$= \frac{dR}{dD}$

$= 175 + 18D + 75D^2$

Put $D = 10$

$= 175 + 180 + 7500$

$= 7855$

03. $y = \log(\sin e^x) + \sqrt{5+x^6}\,.\sec x$. Find dy/dx

SOLUTION :

STEP 1 :

$$\frac{d}{dx}\log(\sin e^x)$$

$$= \frac{1}{\sin e^x}\frac{d}{dx}\sin e^x$$

$$= \frac{1}{\sin e^x}.\cos e^x.\frac{d}{dx}e^x$$

$$= \frac{1}{\sin e^x}.\cos e^x.e^x$$

$$= e^x.\cot e^x$$

STEP 2 :

$$\frac{d}{dx}\sqrt{5+x^6}\,.\sec x$$

$$= \sqrt{5+x^6}\,.\frac{d}{dx}\sec x + \sec x\frac{d}{dx}\sqrt{5+x^6}$$

$$= \sqrt{5+x^6}\,.\sec x.\tan x + \sec x\frac{1}{2\sqrt{5+x^6}}\frac{d}{dx}(5+x^6)$$

$$= \sqrt{5+x^6}\,.\sec x.\tan x + \sec x\frac{1}{2\sqrt{5+x^6}}6x^5$$

$$= \sqrt{5+x^6}\,.\sec x.\tan x + \sec x\frac{3x^5}{\sqrt{5+x^6}}$$

$$= \sec x\left(\sqrt{5+x^6}\,.\tan x + \frac{3x^5}{\sqrt{5+x^6}}\right)$$

STEP 3 :

$$y = \log(\sin e^x) + \sqrt{5+x^6}\,.\sec x$$

$$\frac{dy}{dx} = e^x.\cot e^x + \sec x\left(\sqrt{5+x^6}\,.\tan x + \frac{3x^5}{\sqrt{5+x^6}}\right)$$

SECTION - II

Q4. Attempt ANY **SIX OF** THE FOLLOWING (12)

Q4

01. Find 'x' if Price Index Numbers by Simple Aggregate method is 180

Base year price	12	28	x	26	24
Current year price	38	41	25	36	40

SOLUTION :

$P_{01} = 180$

$$\frac{\Sigma p_1}{\Sigma p_0} \times 100 = 180$$

$$\frac{180}{90 + x} \times 100 = 180$$

$$100 = 90 + x \qquad \therefore x = 10$$

02. 300 students appeared for oral and written test . 180 passed both the test . 90 students failed in both the test . 60 passed in oral but failed in written . Is the data consistent

SOLUTION : A ≡ student passed in oral test

B ≡ student passed in written test

	Written	B	β	TOTAL
oral	A	$(AB) = 180$	$(A\beta) = 60$	$(A) = 240$
	α	$(\alpha B) = -30$	$(\alpha\beta) = 90$	$(\alpha) = 60$
	TOTAL	$(B) = 150$	$(\beta) = 150$	$N = 300$

Comment : Since $(\alpha\beta) < 0$, information is incorrect

03. Find n if $^nP_5 = 42\ ^nP_3$

SOLUTION :

$$\frac{\cancel{n!}}{(n-5)!} = 42 \frac{\cancel{n!}}{(n-3)!}$$

$$\frac{(n-3)!}{(n-5)!} = 42$$

$$\frac{(n-3)(n-4)\cancel{(n-5)!}}{\cancel{(n-5)!}} = 42$$

(DESCENDING ORDER)

$$(n-3)(n-4) = 7.6$$

On comparing

$n = 10$

04. Check the type of association between attributes A and B where

N = 500 ; (A) = 325 ; (B) = 310 ; (AB) = 160

SOLUTION :

1) $\frac{(A) \times (B)}{N} = \frac{\overset{65}{\cancel{325}} \times 310}{\underset{10}{\cancel{500}}} = 201.5$

2) (AB) = 160 ...... given

3) $(AB) < \frac{(A) \times (B)}{N}$

4) there is negative association between attributes A and B

05. Obtain the 3 yearly moving averages for the following data relating to the production of tea in India

Year	:	1941	1942	1943	1944	1945	1946	1947	1948
Production	:	464	515	518	467	502	540	557	571

SOLUTION :

Year	Production	3 YEAR MOVING TOTAL (T)	3 YEAR MOVING AVG (T/3)
1941	464		
1942	515	1497	499
1943	518	1500	500
1944	467	1487	495.67
1945	502	1509	503
1946	540	1599	533
1947	557	1668	556
1948	571		

06. a card is drawn from a pack of 52 cards . What is the probability that it is a face card , given that it is a red card

SOLUTION :

A : face card P(A) = 12/52

B : red card P(A) = 26/52

$A \cap B$: face card and red card = 6/52

E : card drawn is a face card given that it is a red card

$E \equiv A | B$

$P(E) = P(A | B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{6}{26} = {}^{3}/_{13}$

07. In how many ways letters of the word "STORY" be arranged so that

a) T and Y are always together b) T is always next to Y

SOLUTION :

a) T and Y are always together

Consider T & Y as 1 set

1 set of T & Y & 3 other letters can be arranged in ${}^4P_4 = 4!$ ways

Having done that ;

The letters T & Y can then be arranged amongst themselves in ${}^2P_2 = 2!$ ways

By fundamental principle of Multiplication

Total arrangements $= 4! \times 2!$

$= 24 \times 2 = 48$

b) T is always next to Y

Consider T & Y as 1 set

1 set of T & Y & 3 other letters can be arranged in ${}^4P_4 = 4!$ ways

Since T is always next to Y , they SHOULD NOT BE be further arranged in ${}^2P_2 = 2!$ Ways

By fundamental principle of Multiplication

Total arrangements $= 4!$

$= 24$

08. a bag contains 10 white balls and 15 black balls . Two balls are drawn in succession with replacement . What is the probability that first is white and second is black

exp : Two balls are drawn in succession without replacement

E ≡ first is white AND second is black

$E \equiv A \cap B$

$P(E) = P(A \cap B)$

$P(E) = P(A) \times P(B \mid A)$

$= \frac{10}{25} \times \frac{15}{24}$

$= \frac{1}{4}$

Q5. (A) Attempt ANY **TWO OF** THE FOLLOWING (06)

Q5A

01. a problem is given to three students A ,B , C whose chances of solving it are 1/2 , 1/3 & 1/4 respectively . Find the probability that the problem will be solved

SOLUTION :

A : student A can solve a problem $P(A) = 1/2$, $P(A') = 1/2$

B : student B can solve a problem $P(B) = 1/3$, $P(B') = 2/3$

C : student B can solve a problem $P(C) = 1/4$, $P(C') = 3/4$

$E \equiv$ problem is solved

$E' \equiv$ problem is not solved

$E' \equiv A' \cap B' \cap C'$

$$P(E') = P(A' \cap B' \cap C')$$

$$= P(A') \times P(B') \times P(C')$$

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$$

$$= \frac{1}{4}$$

$$P(E) = 1 - P(E')$$

$$= 1 - \frac{1}{4}$$

$$= \frac{3}{4}$$

02. if $P(A) = 1/4$; $P(B) = 2/5$; $P(A \cup B) = 1/2$, then find

a) $P(A \cap B)$ b) $P(A \cap B')$ c) $P(A' \cap B)$

SOLUTION :

a) $$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{1}{4} + \frac{2}{5} - \frac{1}{2}$$

$$= \frac{5 + 8 - 10}{20}$$

$$= \frac{3}{20}$$

b) $$P(A \cap B') = P(A) - P(A \cap B) = \frac{1}{4} - \frac{3}{20} = \frac{5 - 3}{20} = \frac{1}{10}$$

c) $$P(A' \cap B) = P(B) - P(A \cap B) = \frac{2}{5} - \frac{3}{20} = \frac{8 - 3}{20} = \frac{1}{4}$$

03. Two adults and three children are sitting on a sofa and watching TV . Find the probability that the adults are sitting together

SOLUTION :

Exp : two adults and three children are to be arranged on a sofa

$n(S) = {}^5P_5 = 5!$

Event : adults are sitting together

Consider 2 adults as 1 set .

Hence 1 set of 2 adults and 3 children can be arranged amongst themselves in ${}^4P_4 = 4!$ Ways

Having done that ;

The two adults can be arranged in ${}^2P_2 = 2!$ Ways

Hence total ways = 4! x 2!

... BY FUNDAMENTAL PRINCIPLE OF MULTIPLICATION .

$n(E) = 4! \times 2!$

$$P(E) = \frac{n(E)}{n(S)} = \frac{4! \times 2!}{5!} = \frac{\cancel{4!} \times 2!}{5 . \cancel{4!}} = \frac{2}{5}$$

Q5. (B) Attempt ANY **TWO OF** THE FOLLOWING (08)

Q5B

01. Find the cost of Living Index number taking 2001 as base year

Group	p_0	p_1	w	$I = \frac{p_1}{p_0} \times 100$	Iw
A	15	36	60	$\frac{36}{15} \times 100 = 240$	14400
B	48	96	5	$\frac{96}{48} \times 100 = 200$	1000
C	30	90	10	$\frac{90}{30} \times 100 = 300$	3000
D	60	180	15	$\frac{180}{60} \times 100 = 300$	4500
E	45	90	10	$\frac{90}{45} \times 100 = 200$	2000
			$\Sigma w = 100$		$\Sigma Iw = 24900$

$$CLI = \frac{\Sigma Iw}{\Sigma w} = \frac{24900}{100} = 249$$

02. SOLUTION : A ≡ male is literate

B ≡ male is unemployed

	unemployed B	β employed	TOTAL
literate A	(AB) = 4	(Aβ) = 36	(A) = 40
α	(αB) = 8	(αβ) = 152	(α) = 160
TOTAL	(B) = 12	(β) = 188	N = 200

ROUGH WORK

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$= \frac{(4)(152) - (36)(8)}{(4)(152) + (36)(8)}$$

$$= \frac{608 - 288}{608 + 288}$$

$$= \frac{320}{896} = \underline{0.3572}$$

LOG CALC.
2. 5051
– 2. 9523
AL $\bar{1}$. 5528
0. 3572

03. 100 students appeared for two examinations , 60 passed in first examination , 50 passed the second and 30 passed in both. Find the probability that a student selected at random

a) passed in at least one examination   b) passed in exactly one examination

c) failed in both the examination

SOLUTION :

A : student has passed in 'FIRST' exam $P(A) = {}^{60}/_{100}$

B : student has passed in 'SECOND' exam $P(B) = {}^{50}/_{100}$

$A \cap B$: student has passed in 'BOTH' exams $P(A \cap B) = {}^{30}/_{100}$

a) $E \equiv$ student has passed in at least one examination

$E \equiv A \cup B$

$$P(E) = P(A \cup B)$$

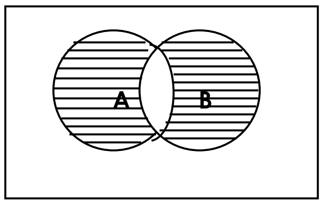
$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{60}{100} + \frac{50}{100} - \frac{30}{100}$$

$$= \frac{80}{100}$$

b) $E \equiv$ student has passed in exactly one examination

$E \equiv (A \cup B) - (A \cap B)$



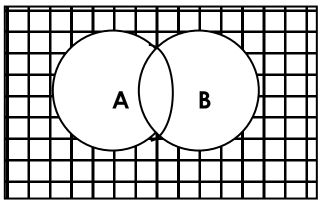
$$P(E) = P(A \cup B) - P(A \cap B)$$

$$= \frac{80}{100} - \frac{30}{100}$$

$$= 50/100$$

c) $E \equiv$ student failed in both the examination

$E \equiv A' \cap B'$



$$P(E) = P(A' \cap B')$$

$$= 1 - P(A \cup B)$$

$$= 1 - 0.80$$

$$= 0.20$$

Q6. (A) Attempt ANY **TWO OF** THE FOLLOWING (06)

Q6A

01. There are 4 professors and 6 students . In how many ways a committee of 4 can be formed so as to include at least 2 professors

SOLUTION :

Case 1 : Committee contains 2 professors and 2 students

This can be done in $^4C_2 \times {}^6C_2$

$= 6 \times 15 = 90$ ways

Case 2 : Committee contains 3 professors and 1 student

This can be done in $^4C_3 \times {}^6C_1$

$= 4 \times 6 = 24$ ways

Case 3 : Committee contains 4 professors and no student

This can be done in $^4C_4 = 1$ way

By FUNDAMENTAL PRINCIPLE OF ADDITION

Total ways of forming the committee = 115

02. Out of 4 officers and 10 clerks in an office , a committee consisting of 2 officers and 3 clerks is to be formed . In how many ways can this be done if one particular clerk must be on the committee

SOLUTION :

since one particular clerk must be on the committee , the remaining 2 clerks have to be selected from the remaining 9 clerks . This can be done in $^9C_2$ ways

Having done that ,

2 officers have to be selected from the 4 officers . This can be done in $^4C_2$ ways

By fundamental principle of Multiplication ,

Total ways of forming the committee $= {}^9C_2 \times {}^4C_2 = 36 \times 6 = 216$

03. $^{n}C_6 : {}^{n-3}C_3 = 33 : 4$ *, find n*

SOLUTION :

$$\frac{^{n}C_6}{^{n-3}C_3} = \frac{33}{4}$$

$$\frac{\frac{n!}{(n-6)!.6!}}{\frac{(n-3)!}{(n-3-3)!.3!}} = \frac{33}{4}$$

$$\frac{\frac{n!}{\cancel{(n-6)!}.6!}}{\frac{(n-3)!}{\cancel{(n-6)!}.3!}} = \frac{33}{4}$$

$$\frac{n!}{6!} \times \frac{3!}{(n-3)!} = \frac{33}{4}$$

$$\frac{n!}{(n-3)!} \times \frac{3!}{6!} = \frac{33}{4}$$

$$\frac{n(n-1)(n-2)(n-3)!}{(n-3)!} \times \frac{3!}{6.5.4.3!} = \frac{33}{4}$$

$$\frac{n(n-1)(n-2)}{6.5.4} = \frac{33}{4}$$

$$n(n-1)(n-2) = 33 . 6 .5$$

$$n(n-1)(n-2) = 990$$

$$n(n-1)(n-2) = 11 \times 10 \times 9$$

On Comparing ; $n = 11$

Q6. (B) Attempt ANY **TWO OF** THE FOLLOWING (08)

Q6B

01. Calculate Fisher's Price Index number

SOLUTION :

p_0	q_0	p_1	q_1	p_0q_0	p_0q_1	p_1q_0	p_1q_1
22	10	25	30	220	660	250	750
34	12	35	40	408	1360	420	1400
28	15	25	25	420	700	375	625
26	15	25	10	364	260	350	250
30	11	35	10	330	300	385	350
				1742	3280	1780	3375
				Σp_0q_0	Σp_0q_1	Σp_1q_0	Σp_1q_1

$$P_{01}(L) = \frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times 100$$

$$= \frac{1780}{1742} \times 100$$

$$= 102.2$$

LOG CALC.
3. 2504
– 3. 2410
AL 0. 0094
1.022

$$P_{01}(P) = \frac{\Sigma p_1q_1}{\Sigma p_0q_1} \times 100$$

$$= \frac{3375}{3280} \times 100$$

$$= 102.9$$

LOG CALC.
3. 5282
– 3. 5159
AL 0. 0123
1.029

$$P_{01}(F) = \sqrt{P_{01}(L) \times P_{01}(P)}$$

$$= \sqrt{102.2 \times 102.9}$$

$$= 102.5$$

LOG CALC.
2. 0094
+ 2. 0123
4. 0217
2
AL 2.0109
102.5

02 Obtain trend line by method of least squares

Year	1959	1960	1961	1962	1963
% insured	11.3	13	9.7	10.6	10.7

SOLUTION :

t	y	u = t – 1961	u^2	yu
1959	11.3	–2	4	–22.6
1960	13	–1	1	–13
1961	9.7	0	0	0
1962	10.6	1	1	10.6
1963	10.7	2	4	21.4
	55.3	0	10	–3.6
	Σy	Σu	Σu^2	Σyu

$$y = a + bu$$
$$\Sigma y = na + b\Sigma u$$
$$55.3 = 5a$$
$$a = 11.06$$

$$yu = au + bu^2$$
$$\Sigma yu = a\Sigma u + b\Sigma u^2$$
$$-3.6 = b(10)$$
$$b = -0.36$$

Hence trend line , $y = 11.06 - 0.36u$ where $u = t - 1961$

03. Without repetition of digits , 4 digit numbers are formed using digits 5 , 6 , 7 , 8 , 9 , 0 . Find the probability that the number formed is ODD and greater than 6000

Exp : 4 digit numbers to be formed using the digits 5 , 6 , 7 , 8 , 9 , 0 .

thousand place can be filled by any of the 5 digits (excluding 0) in $^5P_1$ ways

Remaining 3 places can be filled by any 3 of the remaining 5 digits in $^5P_3$ ways

By fundamental principle of multiplication , $n(S) = {}^5P_1 \times {}^5P_3 = 300$

E : number formed is ODD and greater than 6000

Case 1 : Thousand place is filled by digit 6 , 8

Thousand place can be filled by digit 6 or in 2 way

Since the number is ODD ,

Unit place can be filled by any one of the digits 5 , 7 , 9 in $^3P_1$ ways

Having done that ; remaining two places can be filled by any 2 of the remaining 4 digits in $^4P_2$ ways

By fundamental principle of multiplication ,

nos. formed $= 2 \times {}^3P_1 \times {}^4P_2 = 72$

Case 2 : Thousand place is filled by digit 7 ,9

Thousand place can be filled by digit '7 or 9 in 2 way

Since the number is ODD ,

Unit place can be then be filled by any of the remaining 2 odd digits in 2 ways

Having done that ; remaining two places can be filled by any of the remaining 4 digits in ${}^4P_2$ ways

By fundamental principle of multiplication ,

nos. formed $= 2 \times 2 \times {}^4P_2 = 48$

Therefore ; By fundamental principle of ADDITION , $n(E) = 72 + 48 = 120$

$$P(E) = \frac{n(E)}{n(S)} = \frac{120}{300} = \frac{2}{5}$$